

# Modeling TTL-based Internet Caches

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*Abstract—*

This paper models Internet caches that use time-to-live (TTL)-based caching, the caching mechanism used by Web and DNS caches. In TTL-based caching the cached version of a data item is deemed valid until a certain time (the “time-to-live”) elapses from when the data item is placed in the cache. This simple mechanism scales well to large numbers of caches, but the dependence of hit rates on TTL and access statistics is not clear. To address this, we develop a closed-form formula for the hit ratio based on the simplifying assumption that the sequence of inter-arrival times of the queries for a given data item can be modeled as a sequence of independent and identically distributed random variables. Our model provides an analysis for the cache hit rates of TTL-based Internet caches in terms of the statistics of data accesses and the TTL value set by the origin site of information. Analyzing extensive DNS traces, we find that our model predicts observed statistics remarkably well; in particular, it’s able to adequately explain the somewhat surprising empirical finding of Jung *et al.* [1] that for DNS accesses, the cache hit ratio for the 15 minute TTL is over 80% and increasing the TTL of DNS names from 15 minutes to 24 hours has only a small ( $< 17\%$ ) improvement in the cache hit rate. The prediction is best when we use the observed, empirical distribution of the inter-arrival times of DNS queries. We also find that if the inter-arrival times are modeled according to an analytic distribution, then a Pareto distribution with a point mass yields a better fit than a Weibull distribution, with or without a point mass, or a log Normal, or other candidate distributions.

## I. INTRODUCTION

Caching is one of the oldest techniques for improving performance in computer systems; by storing information locally, caches typically enhance performance by reducing the latency to access information and by reducing the bandwidth required to the origin of information. Internet systems display caching in abundance—the Web and the Domain Name System (DNS) [2] are two important examples of systems that employ caching for these reasons, and it’s believed that they derive significant benefits from doing so.

Caching mechanisms in Internet systems are designed to scale to large numbers of caches. A common way of

achieving scalable caching is to use *time-to-live (TTL)-based caches*, which work as follows. For any data item  $D$ , the site that maintains the current, authoritative version of  $D$  is called the *origin site*. If the origin receives a request for  $D$  at time  $t$  from a requestor (typically a cache shared by many clients, or the cache on a client itself), it returns the current version along with a TTL value,  $T$ . The requestor is allowed to cache  $D$ . Any subsequent requests made to the requestor by clients in the time interval  $(t, t+T)$  can be served from the requestor’s cache *without* contacting the origin site. However, the first request after time  $t + T$  *must* go to the origin site, since the TTL has expired for  $D$  in the requestor’s cache.

TTL-based caching scales well because origin sites don’t have to maintain any per-requestor (*i.e.*, per-cache) state, nor even know of the existence of caches. This also enables “opportunistic caching” by caches across the Internet, since they don’t need to inform the origin that they are caching data. The trade-off in TTL-based caching is between consistency and scalability—because the origin does not invalidate cached content, clients may occasionally retrieve outdated data until the previously advertised TTL expires.

A significant number of cache misses in Internet systems that employ TTL-based caching occur because of TTL expiration. For DNS caches, essentially all misses occur because of TTL expiration or first access to a domain name and capacity misses are negligible due to an ample storage in comparison to a small data item. This is also true for Web data that change frequently with time (e.g., news headlines, sports scores, tickers, etc.). Reflecting common workload patterns typical in Web and DNS accesses, we only consider read-only data in this paper. These caches don’t exhibit “conflict” misses caused by multiple concurrent writes.

Several researchers have conducted experimental studies of caching performance in the Web [3], [4], [5], [6], [7] and more recently, DNS [1], [8]. In many of these cases, an increasing rate of the cache hit ratios is surprisingly slow as TTL values gets larger; for example, in an extensive trace-driven study of DNS cache performance driven by large client-side TCP and DNS traces [1], we found that the cache hit rate does not improve by more than 17% even when the TTL is increased from 15 minutes to 24

hours! We found this surprising—it seems reasonable to expect many more accesses to any given origin site in 24 hours than in 15 minutes, and since (only) the first access after the TTL expires misses in the cache, the small difference in hit rates for the large increase in TTL is rather counter-intuitive.

We speculated that this incremental improvement in cache hit rates had to do with the nature of accesses to the origin site from the clients sharing a cache. Motivated by this observation, we seek to answer the following fundamental question in this paper: *How does the cache hit rate for TTL-based Internet caches depend on the statistics of data accesses and the TTL values of data items?*

Somewhat surprisingly, we find little previous research (except recent work by Cohen *et al.* [6], [7], discussed in Section II) devoted to answering this question, despite its importance in understanding how well Web and DNS caches work. Building on the observation that the first access after TTL expiration incurs a cache miss, while all subsequent accesses until the next expirations hit in the cache, we develop an analytic model for the cache hit ratio based on a renewal process that answers the above question. We then describe a computationally tractable way to come up with numerical solutions for the model and show that our model predicts hit ratios remarkably well. An important component of our model is the inter-arrival distribution of accesses to the cache for a given, arbitrary data item, for which we find that among the several models we considered, a Pareto distribution with a point mass is a good fit.

In Section II we survey related work. Section III describes our analytic model and Section IV presents numerical simulations that show that our model predicts observed cache hit rates for three different datasets accurately. We conclude with a summary of our findings and suggestions for future work in Section V.

## II. RELATED WORK

### A. Analytic Models of Cache Hit Rates

There has been a good deal of research attention on determining hit ratios at caches as a function of various cache replacement policies, such as *least recently used* (LRU) and *first in first out* (FIFO) [9], [10].

In contrast, there has been relatively little work on hit ratios as a function of the time-to-live parameter and the request arrival process. Focusing on the impact of aging on caching performance, Cohen and Kaplan developed a simple cache hit model for three specific inter-request time distributions — fixed-frequency, Poisson, and Pareto, and derived the miss rates under different cache configura-

tions. In their work, a cache may obtain a data item from other caches than the origin site [6].

In the subsequent work [7], Cohen *et al.* further explored the aging issue and demonstrated the relation of the miss rate at a client cache to the distribution of TTL across data sources. When a source can perform *pre-term refresh* or *extend lifetimes* of a data item, they presented analytic results of the miss rate for a Poisson and fixed-frequency inter-request time distribution.

We do not consider a multi-level cache structure in this paper. Rather we consider a single cache where a data item is always fetched from the origin, and focus on analytic models. In comparison with Cohen *et al.*, our model provides a cache hit probability for any arbitrary inter-request time distributions.

### B. Analytic Models of TCP Connection Inter-Start Processes

In this study, we use a TCP connection arrival process to model a cache reference. This approach was first introduced in [1] in trace-driven simulations of DNS cache behavior based on the observation that TCP was a major driving application of DNS lookups. This method allows us to estimate DNS cache hit rate varying the TTL and the degree of cache sharing using empirical distributions for the inter-query times. The possible causes of errors by using TCP connections instead of real DNS lookup traces are also addressed in [1]. In the same paper, the authors made an initial attempt to explain an asymptotic behavior of DNS hit rate with a renewal assumption and showed that a heavy-tailed Pareto distribution was a good fit for the distribution for TCP connection inter-arrivals. Extended from that work, we include a Weibull distribution to seek a good fit for an analysis, as suggested by Feldmann [11]. Feldmann studied the TCP connection arrival process for the aggregation across all destinations and calculated maximum likelihood estimators for the parameters of fitted distributions. In contrast, we focus on a TCP connection arrival process for a given, arbitrary destination and use the `fminsearch` function in `matlab` [12], which finds a good fit using an unconstrained nonlinear optimization [13].

## III. ANALYTIC MODELS

In this section we present a simple, tractable analytic model for the query process to a cache and obtain a closed-form formula for the hit ratio at the cache as a function of the TTL parameter. The section concludes with a discussion of the numerical computation of the hit ratio obtained from the analytic model, as well as from the trace-driven simulations.

### A. Renewal Model Query Process

The model considers a given, arbitrary data item (e.g., a DNS name or a Web object) and the sequence of queries to a cache for that data item. We make the simplifying assumption that the sequence of inter-arrival times of queries for the given data item can be modeled as a sequence of independent and identically distributed (i.i.d.) random variables. The i.i.d. assumption is equivalent to assuming that the inter-query times are a renewal process. Indeed, reality is more complicated. One might expect that the process of inter-query times is “bursty”, and has positive autocorrelation. Thus, heuristically one would expect that the true hit ratio should be higher than that predicted by a model based on an i.i.d. assumption. That is, one would expect that the i.i.d. assumption is conservative, possibly very conservative, from the perspective of predicting how high is the hit ratio. However, as shown in Section IV-C, in spite of the simplification, the resulting model predicts the hit ratio quite well for the obtained datasets. In particular, Paxson and Floyd pointed out that i.i.d. Pareto interarrivals were useful as approximations to particular finite process arising in wide-area network traffic [14]. Feldmann also observed that i.i.d. Weibull model captured well the burstiness of Web connection interarrivals [11]. We believe that both cases show that using i.i.d. assumption for the inter-query times closely models the underlying query arrival process of Internet caches even if it is not statistically exact.

Let  $X_i$  be the time interval between the  $i - 1^{th}$  query for the given data item and the  $i^{th}$  query. Assume  $X_i$  is a proper random variable:  $X_i$  has a distribution function  $F(x) \equiv \Pr(X_i \leq x)$ , where  $\lim_{x \rightarrow \infty} F(x) = 1$  (as opposed to a value less than 1). Let time  $t = 0$  be chosen at the arrival of a query to the cache for which the data item is not cached, or the TTL has expired, i.e. the time of a cache miss.

$X_0 = 0$  and  $X_i$ 's are proper, non-negative, i.i.d. random variables,  $X_i$  may have infinite mean.

Let  $N(t)$  equal the number of queries for the given data item in the interval  $(0, t]$ .  $N(t)$  is called the renewal counting process. Note that the event at  $t = 0$  is excluded. Let  $S_n = X_1 + X_2 + \dots + X_n$  ( $S_0 = 0$ ), and

$$\begin{aligned} \Pr(S_n \leq t) &= \Pr(X_1 + X_2 + \dots + X_n \leq t) \\ &= F^{(n)}(t), \end{aligned} \quad (1)$$

where  $F^{(n)}(t)$  denotes the  $n^{th}$ -fold convolution of the distribution function  $F(x)$  with itself.

Let the value of the time to live (TTL) parameter be  $T$ .

**Key observation:** The renewal counting process evaluated at time equal to the TTL,  $N(t) |_{t=T}$ , models the

number of cache hits per cache miss, for the given data item.

Figure 1 illustrates the idea. At time  $t = 0$ , there is a cache miss. Subsequently, three queries occur, at times  $S_1, S_2, S_3$ , before the TTL expires at time  $t = T$ . These three queries are cache hits. The subsequent, fourth query at time  $S_4$  occurs after  $t = T$  and is a cache miss. Thus in Figure 1,  $N(T) = 3$  and the number of cache hits per miss is 3. Note that one could reset the time origin to  $S_4$  and the resulting process would be stochastically equivalent to the first.

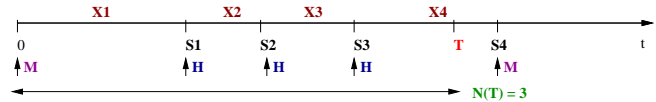


Fig. 1. Time line diagram of queries to a given data item, and associated model random variables.

In summary, using a renewal assumption, we obtain a renewal counting process, which when evaluated at a particular time period, the TTL, models the number of cache hits per miss. This random variable in turn can be used to compute other quantities of interest. In Section III-B, its expectation is used to compute the hit and miss ratios.

**Selection of inter-query time distribution:** Given the renewal assumption, the remaining free attribute of the model is the distribution for the inter-query time,  $F(x)$ . One natural choice is the empirical distribution obtained from the collected data. As the model is based on the viewpoint of picking a given, randomly selected data item, conceptually, one computes the empirical distribution for each data item of interest (say each of the data items in the data set) and then takes a weighted average of the distributions, where the weights are the fraction of queries for a given data item. (An equivalent and more straight forward, calculation is to compute the relative frequency of the inter-query times to all of the data items of interest.) Another natural choice for the distribution is an analytic one, such as Pareto or Weibull. Section IV considers both empirical and analytic distributions.

### B. Renewal Model for Hit and Miss Ratios

In the context of the inter-query renewal model, we derive an expression for the long-term ( $t \rightarrow \infty$ ) hit and miss ratios as a function of the TTL,  $T$ . In particular, let  $H(T)$  denote the limiting hit ratio, which is defined as the number of queries that are a cache hit divided by the total number of queries, for a given data item, as  $t \rightarrow \infty$  and given the TTL is  $T$ . The limiting miss ratio, denoted  $M(T)$ , is analogous, where “cache hit” is replaced with “cache miss.”

*Theorem 1:* If the inter-query times  $X_i$ 's to a given data item are proper, non-negative, independent and identically distributed random variables, whose mean may be infinite, then

$$H(T) = \frac{E[N(T)]}{E[N(T)] + 1} \text{ and}$$

$$M(T) = \frac{1}{E[N(T)] + 1} \text{ with probability one. (2)}$$

**Remark:** For any finite time  $t$ , the hit and miss ratios have a complicated distribution. However, due to the strong law of large numbers, in the limit as  $t \rightarrow \infty$ , the distribution simplifies to a single point mass.

Equation (2) is heuristically appealing: the hit ratio equals the mean number of hits per miss divided by the same quantity plus the one cache miss. Equivalently, if one thinks of an episode, or cycle, of a cache miss followed by cache hits (if any) before the TTL expires, then Equation (2) says that the hit ratio equals the mean number of hits in a cycle divided by the mean number of queries (one miss plus hits).

The proof of the theorem is standard, with a few subtle points, and is given in the appendix.

### C. Calculation of Hit Ratio

The following two sections provide background on the calculation of the hit ratio in respectively the renewal model and the trace-driven simulation, and are used in computing the numerical results in Section IV-C.

1) *Calculation of Hit Ratio in Renewal Model:* The computation of the hit ratio in the renewal model, Equation (2), requires the computation of  $E[N(T)]$ , the mean number of queries over the interval  $(0, T]$ . The expectation of the renewal counting process is a common entity of interest in renewal theory and is called the renewal function:  $m(t) \equiv E[N(t)]$ .

$m(t)$  can be expressed as

$$m(t) = \sum_{n=0}^{\infty} F^{(n)}(t), \quad t \geq 0$$

where  $F^{(n)}(t)$  denotes the  $n^{\text{th}}$ -fold convolution of the distribution function  $F(t)$  (Equation (1)) [15].

Note that the hit ratio,  $H(T)$  from Equation (2), can be written as a function of  $F^{(n)}(t)$ :

$$H(T) = \frac{\sum_{n=0}^{\infty} F^n(T)}{\sum_{n=0}^{\infty} F^n(T) + 1} \quad (3)$$

Although (3) is of conceptual interest as it expresses the hit ratio in terms of the basic, underlying entity of the renewal process, the inter-arrival distribution,  $F(x)$ , it is not a convenient form for numerical computation.

The renewal function,  $m(t)$  satisfies the *renewal equation*:

$$m(t) = F(t) + \int_0^t m(t-x)dF(x) \quad (4)$$

Discretizing Equation (4) yields a numerically convenient iteration for  $m(t)$ , see [15] for details.

2) *Calculation of Hit Ratio in Trace-Driven Simulation:* In a trace-driven simulation, the computation of the hit ratio is straightforward: simply count the number of queries and the number that are cache hits. However, this requires repeating the counting process for all data items in the trace, which is not computationally efficient, especially when it takes long to scan through a data trace. To avoid this, one can also compute the hit ratio in a way that mirrors the renewal model and provides sample path realizations for the entities that the stochastic model attempts to describe. We describe this latter calculation in this section.

Define a *cycle* as the sequence of a cache miss followed by cache hits (if any) before the TTL expires, for a given data item. Let  $C$  be the number of such cycles in the simulation run<sup>1</sup>. Every query is in one and only one of these cycles.

Let  $f_n$  be the number of cycles in the simulation run in which there were  $n$  cache hits ( $n = 0, 1, 2, \dots$ ). Since every cycle contains one cache miss,  $f_n$  also equals the number of cycles in the simulation run in which there were  $n + 1$  queries. Thus, the total number of cache hits in the simulation can be expressed as  $\sum_{n=0}^{\infty} n \cdot f_n$  and the total number of cache queries (hit + miss) can be expressed as  $\sum_{n=0}^{\infty} (n + 1) \cdot f_n$ .

Note that there is an edge effect at the end of the simulation: multiple cycles are likely to be in progress. One could view this as negligible noise if the simulation runs over a period of time much longer than the TTLs. Alternatively, one could omit the in-progress cycles at the end of the simulation from the counters,  $f_n$ 's and  $C$ .

The hit rate is defined to be the number of cache hits divided by the number of cache queries in the simulation run. So, hit rate can be expressed as:

$$H = \frac{\sum_{n=0}^{\infty} n \cdot f_n}{\sum_{n=0}^{\infty} (n + 1) \cdot f_n} \quad (5)$$

<sup>1</sup>Note at a given point in time, multiple cycles can be in progress

From the definitions,

$$\begin{aligned} \frac{f_n}{C} &= \text{fraction of cycles in which there are } n \\ &\quad \text{cache hits} \\ &= \text{fraction of cycles in which there are} \\ &\quad n + 1 \text{ queries} \\ \sum_{n=0}^{\infty} \frac{f_n}{C} &= 1. \end{aligned}$$

The above quantities provide sample path estimates for the modeled renewal counting process,  $N(t)$  evaluated at  $t = T$ , the TTL. Recall that  $N(T)$  is a random variable representing the number of cache hits in a randomly chosen cycle for a random data item, given the TTL is  $T$ . Based on the trace-driven simulation:

$$\Pr(N(T) = n) = \frac{f_n}{C}$$

The sample mean of  $N(T)$ , denoted  $\overline{N(T)}$ , is

$$\overline{N(T)} = \sum_{n=0}^{\infty} n \cdot \frac{f_n}{C}. \quad (6)$$

The hit ratio, Equation (5), can be expressed in terms of  $\overline{N(T)}$ . Dividing the numerator and denominator of (5) by  $C$  and substituting in (6) yields:

$$H = \frac{\sum_{n=0}^{\infty} n \cdot \frac{f_n}{C}}{\sum_{n=0}^{\infty} n \cdot \frac{f_n}{C} + \frac{f_n}{C}} = \frac{\overline{N(T)}}{\overline{N(T)} + 1} \quad (7)$$

In summary, for computing the hit ratio in the trace-driven simulation one can directly count the number of queries and the number of cache hits. A numerically equivalent calculation is to use Equation (5), which in turn is equivalent to (7). None of these calculations make any i.i.d assumptions about random variables.

One can then compare these results with a calculation that does make the i.i.d. assumption: calculate the empirical distribution for the inter-query time  $F(x)$  and apply the iteration of Equation (4) and substitute into Equation (2). One can also use Equations (4) and (2) to compute the hit ratio for candidate analytic inter-query time distributions. The next section reports on such calculations.

#### IV. NUMERICAL RESULTS

The previous section described an analytic model of the hit ratios for TTL-based Internet caches. This section begins with a discussion on the analytic models of a TCP connection process which generates DNS queries, thus providing a useful estimator for  $F(x)$  in Section III-A.

Using DNS as an example system, we then present the hit ratios calculated in three different ways — using trace-driven simulation, using the renewal assumption with the empirical distribution of  $F(x)$  obtained from the data set, and using the renewal assumption with an analytic distribution. Finally, we evaluate our analytic model developed in Section III by comparing the above hit ratios and exploring the gap resulting from the renewal assumption and an approximate model of  $F(x)$ .

##### A. The Data

We use three separate traces collected at the border gateway of MIT's Laboratory for Computer Science (LCS) and Artificial Intelligence Laboratory (AI) and at a link that connects Korea Advanced Institute of Science and Technology (KAIST) to the rest of the Internet. The first trace, `mit-jan00` was collected from 3 January to 10 January 2000; the second, `mit-dec00` was collected from 4 December to 11 December 2000. Both were collected at the same point in MIT. The third set, `kaist-may01` was collected at KAIST from 18 May to 24 May 2001. Each data trace recorded over 3 million TCP outgoing connections generated from over 900 clients for over 30 thousands different destinations. A detailed description of the traces is available in [1].

With the same data sets, Jung *et al.* [1] estimated the DNS cache hit rates inside the traced networks by trace-driven simulations. Assuming that TCP was a major application that drives DNS lookup sequences, they used TCP connections to model a cache reference, and conducted simulations showing the impact of a time-to-live parameter on the DNS cache hit ratio. In this section, we evaluate our renewal model by calculating the hit ratio using the methodology discussed in Section III-C.1 and compare it with the one from the trace-driven simulations where the hit ratio is simply the number of cache hits divided by the number of queries. Computing the hit ratio via Equations (4) and (2) requires the inter-query time distribution,  $F(x)$ , which can be obtained either empirically or analytically from a given data trace.

To deduce the distribution of the inter-query time at a cache from the real traffic, we calculate the time difference between two consecutive connection arrivals for a given destination IP address, denoted as  $x$ , and count the frequency of  $x$  across all pairs of such occurrences. For all three traces,  $x$  spans 9 orders of magnitude, ranging from  $10^{-3}$  to  $10^6$  seconds. It is also noticeable that there is a jump at time 1 ms which suggests that there are a number of connections back-to-back in very close succession.

Table I lists the statistics of each data set including the median, the mean,  $E[x]$ , the 95<sup>th</sup>-percentile, and the stan-

standard deviation,  $\sigma_x$ . Due to a heavy tail,  $E[x]$  and  $\sigma_x$  are skewed by large values. For instance,  $E[x]$  is more than 400 times larger than the median. To reduce the scale, we transform data using a natural log, and the corresponding mean,  $E[\ln x]$  and standard deviation,  $\sigma_{\ln x}$ , are listed in Table I.<sup>2</sup>

TABLE I  
STATISTICS FOR TCP CONNECTION INTER-START TIMES FOR A GIVEN DESTINATION

	mit-jan00	mit-dec00	kaist-may01
Median	4 (sec)	7 (sec)	1 (sec)
$E[x]$	1977 (sec)	2814 (sec)	325 (sec)
95 <sup>th</sup> -percentile	3913 (sec)	5932 (sec)	173 (sec)
$\sigma_x$	14348 (sec)	19587 (sec)	5024 (sec)
$E[\ln x]$	9.0 (msec)	9.1 (msec)	6.7 (msec)
$\sigma_{\ln x}$	3.9 (msec)	3.8 (msec)	3.3 (msec)

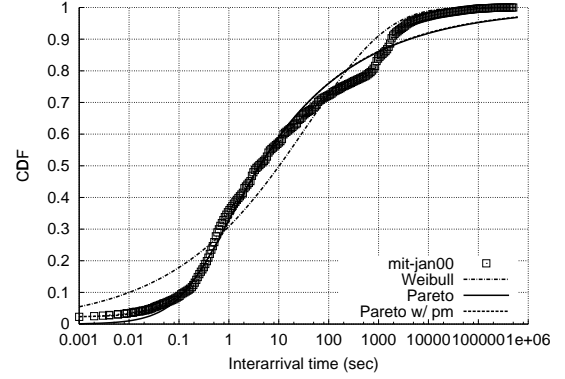
B. Analytic Models of TCP Connection inter-start Time

In this section we derive analytic models describing the distributions for TCP connection inter-start time for three data sets, which is a good approximation for and can substitute for the analytic inter-query time distribution,  $F(x)$ , defined in Section III-A.

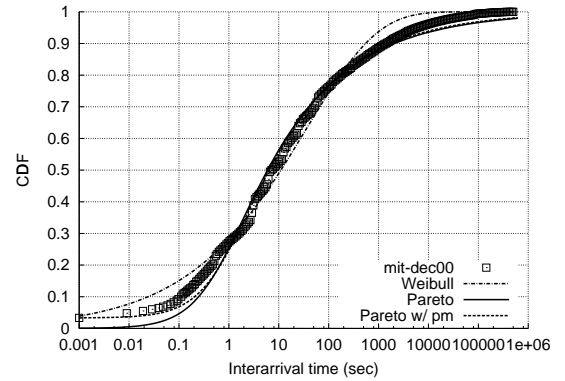
A number of well-known probability distributions were considered as candidates for the distribution function  $F(x)$  for each data trace, including an exponential, a Normal, a Pareto, a Weibull, a log Normal, a log Pareto, a Pareto with a point mass, and a Weibull with a point mass. For the sake of brevity, we report the results for the three best performing choices — a Weibull, Pareto, and Pareto distribution with a point mass, all of which capture a heavy tail feature of the TCP connection inter-start time distribution. Parameter estimation is done with `matlab` using an unconstrained nonlinear optimization [12]. Figure 2 illustrates the fitted distributions and estimated parameters along with the empirical cumulative distribution, shown as square dots.

The fitted Weibull,  $W(x)$ , captures a spike at  $t = 0.001$  (sec) while the fitted Pareto,  $P(x)$ , fits well starting from  $t = 0.1$  (sec). The fitted Pareto, however, estimates a heavy tail better than the Weibull, whose distribution has a decreasing exponential term. As a result, the Weibull

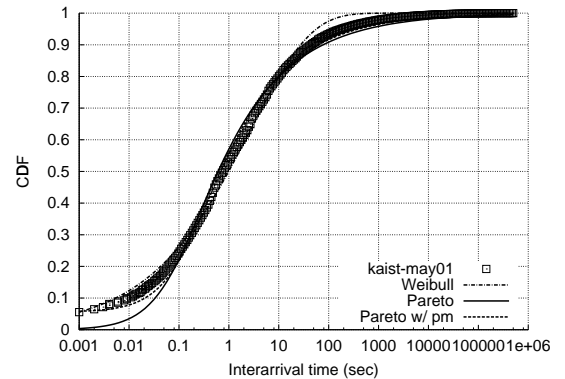
<sup>2</sup> $x$  is measured in a granularity of millisecond and the smallest value is 1 (ms) for the log transformation.



(a) mit-jan00 (Weibull:  $d = 39227$ ,  $c = 0.27$ ; Pareto:  $a = 0.23$ ,  $k = 223$ ; Pareto w/ pm:  $a = 0.24$ ,  $k = 274$ ,  $w = 0.023$ )



(b) mit-dec00 (Weibull:  $d = 36306$ ,  $c = 0.31$ ; Pareto:  $a = 0.28$ ,  $k = 540$ ; Pareto w/ pm:  $a = 0.29$ ,  $k = 735$ ,  $w = 0.033$ )



(c) kaist-may01 ( Weibull:  $d = 2729$ ,  $c = 0.36$ ; Pareto:  $a = 0.34$ ,  $k = 92$ ; Pareto w/ pm:  $a = 0.37$ ,  $k = 153$ ,  $w = 0.056$ )

Fig. 2. Fitted Weibull  $W(x) = (1 - e^{-\frac{x}{d}})^c$ , Pareto  $P(x) = (1 - (\frac{k}{x+k})^\alpha)$ , and Pareto distribution with a point mass  $P(x) = w + (1 - w) \times (1 - (\frac{k}{x+k})^\alpha)$  for TCP connections inter-arrivals

distribution approaches 1 faster than the empirical distribution.

With a point mass at  $t = 0.001$  (sec), the second fitted Pareto,  $\tilde{P}(x)$ , results in a better fit *both* for the beginning and the tail as shown in Figure 2.

This observation can be confirmed using the discrepancy measure,  $\hat{\lambda}^2$  [16] [17].  $\hat{\lambda}^2$  is a modified chi-squared test that enables us to compare discrepancies for different values of the number of bins. Paxson [17] uses the result of Scott [18] in modeling wide-area connection inter-arrival. With fixed-sized bins the mean-square error is minimized with a bin width given by  $w = 3.49\sigma_x n^{-1/3}$ , where  $n$  is the number of samples. Both  $n$  and corresponding  $w$  are listed in Table II.

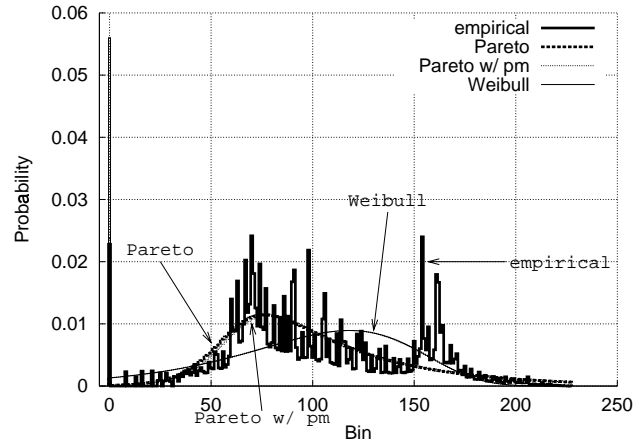
TABLE II  
BIN-WIDTH  $w$  TO MINIMIZE ERROR IN APPROXIMATING A  
DISTRIBUTION USING FIXED-SIZED BINS

	mit- jan00	mit- dec00	kaist- may01
$n$	3571746	4483468	6304553
$w$	0.089	0.080	0.062

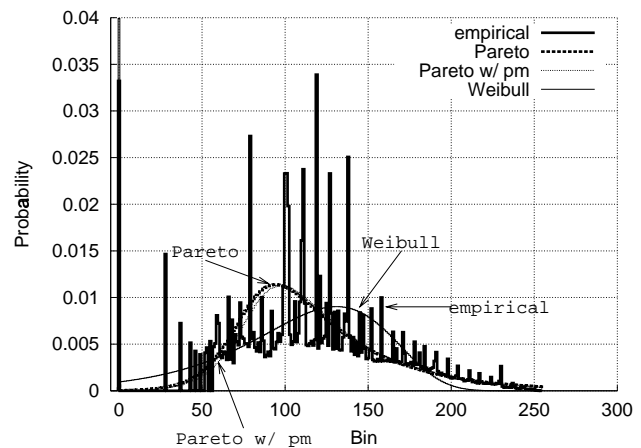
Table III shows the goodness-of-fits for three different fitted distributions with fixed-size bins where a bin size is determined as shown in Table II. Lower values indicate a better fit.<sup>3</sup> For all data sets, the fitted Pareto with a point mass yields a better match to the empirical distribution than the other two distributions.

Complementing the plots of the distribution functions and the numerical goodness-of-fit results, Figure 3 provides a visual comparison via a histogram of the data sets along with a histogram calculated from the three fitted analytic distributions. First, we transform the  $x$  value into a log scale and count the number of samples falling into each range of size  $w$  from the empirical data. For an interval  $(x_1, x_2]$ , a histogram is calculated using the cumulative distribution's values at  $x_2$  and  $x_1$ . Frequent spikes and the shape of humps existing in the empirical data make it hard to yield a good fit over the entire range. In particular, as the fitted Weibull distribution approaches 1 faster than the empirical distribution, for the larger values of  $x$  the difference in the cumulative distribution's values is small; and consequently a very small denominator of the  $\hat{\lambda}^2$  results in a huge discrepancy for kaist-may01 trace.

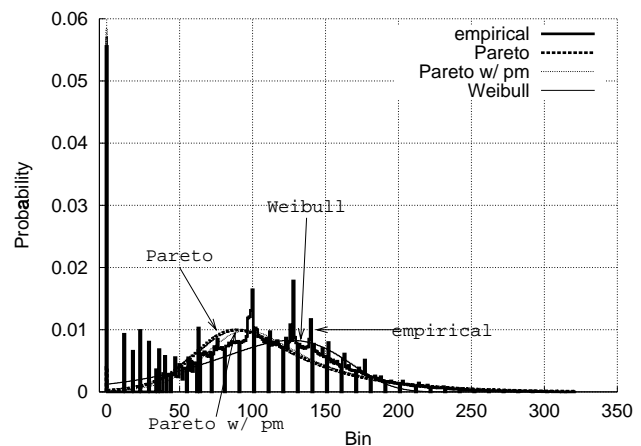
<sup>3</sup>We use  $\langle_\sigma$  operator as described in [17]



(a) mit-jan00



(b) mit-dec00



(c) kaist-may01

Fig. 3. A histogram of the data sets vs. a histogram of the numerical goodness-of-fit results (width =  $w$ )

Instead of dividing  $x$  range into equal-sized bins, we

use an alternate approach that uses an equal-sized bin over the cumulative distribution (CDF) range. From the empirical distribution, we seek the largest CDF value of each bin and keep track of the corresponding  $x$  values.

Note that at jumps in the empirical distribution, a bin can have no sample points, and therefore doesn't give the associated  $x$  value; in this case, the bin is merged with the next adjacent bin until there appears a bin with some sample points. The histogram, is then the differential of the CDF values calculated from the two adjacent  $x$  values picked from the previous scanning process.

Figure 4 compares the histograms of the empirical and analytic distributions when the CDF range is partitioned into 200 equal-sized bins. With bins having roughly equal probability mass, the goodness-of-fit gets smaller for all cases (See Table III and Table IV for comparison). It also confirms that the fitted Pareto distribution with a point mass yields the best fit of all three distributions. However, note that the parameters of the fitted Pareto differ markedly for the different data sets (Figure 2). Thus, there is no Pareto with fixed, selected parameters that uniformly fits well across our data sets.

TABLE III

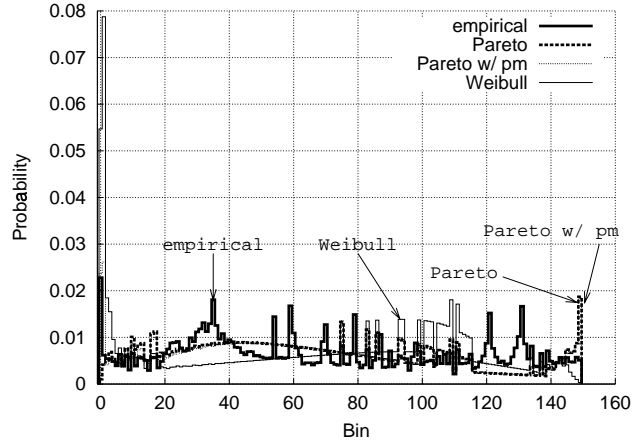
INTERVAL  $[(\hat{\lambda}^2 - \sigma_\lambda), (\hat{\lambda}^2 + \sigma_\lambda)]$  FOR THE FITTED MODELS WHERE  $w$  IS THE WIDTH OF THE BINS

	mit-jan00	mit-dec00	kaist-may01
Weibull	0.945 - 0.952	33.142 - 34.674	364569724 - 393066504
Pareto	1.088 - 1.096	2.968 - 2.991	1.262 - 1.269
Pareto w/ point mass	0.689 - 0.693	1.283 - 1.295	0.790 - 0.796

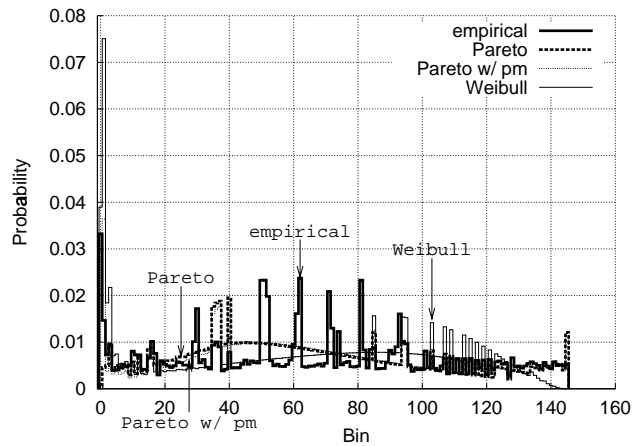
TABLE IV

INTERVAL  $[(\hat{\lambda}^2 - \sigma_\lambda), (\hat{\lambda}^2 + \sigma_\lambda)]$  FOR THE FITTED MODELS WHERE BIN SIZE IS DETERMINED BY FIXED  $y$  INTERVAL (WIDTH = 0.005)

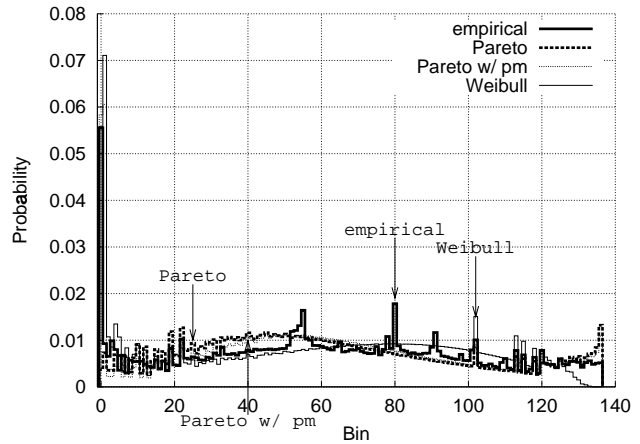
	mit-jan00	mit-dec00	kaist-may01
Weibull	0.742 - 0.746	6.250 - 6.366	14248- 14568
Pareto	0.928 - 0.936	2.396 - 2.418	0.842 - 0.847
Pareto w/ point mass	0.498 - 0.494	0.326 - 0.328	0.162 - 0.163



(a) mit-jan00



(b) mit-dec00



(c) kaist-may01

Fig. 4. A histogram of the data sets vs. a histogram of the numerical goodness-of-fit results (fixed  $y$  width = 0.005). Note that the number of bin with non-zero sample points is less than  $1/0.005 = 200$  for all three cases.



### C. Comparison of Hit Ratios

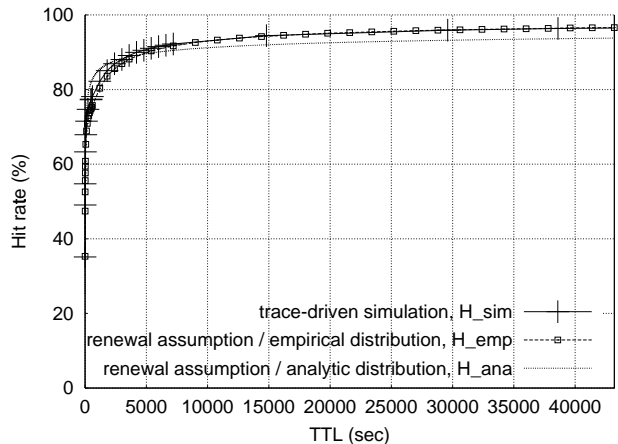
Figure 5 compares cache hit ratios as a function of TTL obtained by three different methodologies. A solid line plots the hit ratio,  $H_{sim}$ , from trace-driven simulations, and  $H_{sim}$  is calculated exactly in the same way described in [1]. A dotted line with a square represents the hit ratio,  $H_{emp}$ , in (Equation (2)) calculated using the empirical distribution of the inter-query time,  $F(x)$ , from each trace. Lastly, a dotted line shows the hit ratio,  $H_{ana}$  in (Equation (2)) calculated using the fitted Pareto distribution with a point mass, which describes an inter-query process with the smallest discrepancy of three candidate models, as shown in Table III and IV.

Comparing  $H_{sim}$  and  $H_{emp}$ , we found that the renewal model worked surprisingly well. The renewal model closely follows the simulation results and the difference is less than 2% up to TTL equal to 86400 (sec), which suggests the inaccuracy of the i.i.d. simplifying assumption is remarkably small with regard to the DNS query process itself. The following example shows how the renewal model explains the empirical findings observed in [1].

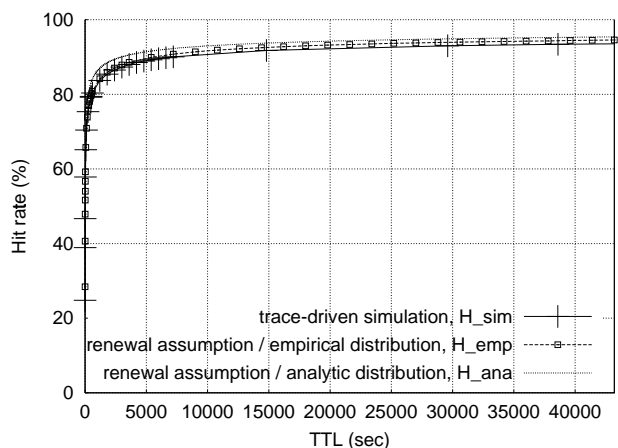
Note that for a TTL of 15 minutes (900 seconds), the hit ratio is relatively high — over 80% for mit-jan00 and mit-dec00, and over 94% for kaist-may01. This leaves room for only modest increase in the hit ratio for larger TTL's. For example, for a TTL of 24 hours (86400 seconds), the hit ratios are 97% for mit-jan00, 94% for mit-dec00, 98% for kaist-may01, respectively. From Equation (2), the hit ratio will be 80% when the mean number of cache hits per miss,  $E[N(T)]$ , is 4. A nominal sample path would be 4 inter-query times before the TTL expires. However, note that the mean for just one inter-query time is greater than 15 minutes, for the MIT traces (Table I). It is the high variability of the inter-query times that enables  $E[N(T)]$  to be as big as 4 (and the hit ratio as high at 80%).

If, in contrast, the inter-query times were rather regular (in the extreme deterministic), but, with the same mean, then for  $T = 15$  minutes,  $E[N(T)]$  would be less than 1 and the corresponding hit ratio would be less than 50%, and decreasing to 0% for the case of deterministic inter-query times. For instance, if the inter-query times were exponentially distributed (i.e., a Poisson process) with mean of 2,000 seconds (corresponding to the MIT traces), then for a TTL of 15 minutes,  $E[N(T)] = 900/2000 = 0.45$ , and the hit ratio would be only 31%.

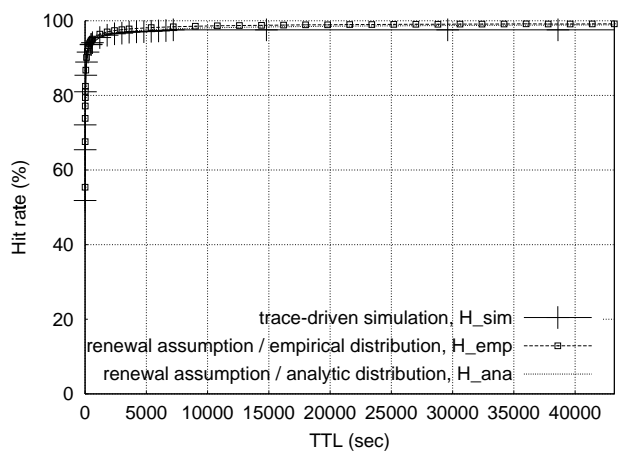
However, when an approximate analytic model of  $F(x)$  replaces the empirical distribution of the inter-query time, the resulting hit ratio  $H_{ana}$  is less accurate.



(a) mit-jan00



(b) mit-dec00



(c) kaist-may01

Fig. 5. Comparing DNS hit rates obtained by three different methods

The inaccuracies are due to the complicated structure of the real inter-query time distribution, which cannot be

described by the fitted model (Figure 3 and 4). But, for all three data sets,  $H_{ana}$  by the fitted Pareto with a point mass predicts the hit ratio better than the two other distributions discussed in Section IV-B.

## V. CONCLUSION

This paper has presented analytic models for the cache hit ratio for TTL-based Internet caches in terms of the statistics of data accesses and the TTL value set by the origin site of information. We developed a closed-form formula for the hit ratio based on the renewal assumption where the sequence of inter-arrival times of the queries for a given data item can be modeled as a sequence of independent and identically distributed random variables.

Analyzing extensive DNS traces, we find that our model predicts observed statistics remarkably well. In particular, it's able to adequately explain the somewhat surprising empirical finding of Jung *et al.* [1] that for DNS accesses, the hit ratio for the 15 minute TTL is over 80% and increasing TTL from 15 minutes to 24 hours has only a small improvement in the cache hit ratio. The prediction is best when we use the observed, empirical distribution of the inter-arrival times of DNS queries. We also find that if the inter-arrival times are modeled according to an analytic distribution, a Pareto distribution is a better fit than a Weibull distribution.

In this paper, we considered a single cache shared by entire clients but our model can be easily extended to the multi-level cache structure where the TTL of a data item is drawn from a certain distribution. In on-going work, we are investigating the renewal-model prediction of hit ratios varying the degree of client sharing as opposed to all sharing a cache. Also, in light of the good fit of the hit ratio function, we are examining the inaccuracy of the i.i.d. simplifying assumption with regard to the query process itself.

## ACKNOWLEDGMENTS

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## APPENDIX

This section provides a proof of the theorem in Section III-B.

Define a *cycle* as the sequence of a cache miss followed by cache hits (if any) before the TTL,  $T$ , expires, for a given data item. The cycle starts at the cache miss. The length of a cycle is defined as the time interval from the start of the cycle until the start of the next cycle. Figure 1 in Section III-A illustrates a cycle in which there are three cache hits, and whose length is  $S_4$ . The sequence of cycle start times constitutes a renewal process, where the inter-start time has the same distribution as  $\sum_{i=1}^Y X_i$ , where the random variable  $Y$  is the smallest integer such that  $\sum_{i=1}^Y X_i > T$ .

Let  $t = 0$  be the start time of the first cycle, and index the cycles  $i = 1, 2, 3, \dots$ . Let  $C(t; T)$  denote the number of cycles to have completed over the interval  $(0, t]$ , given that the TTL is  $T$ . Since the inter-query times,  $X_i$ 's, are assumed to be a proper random variable, and the property of "proper" is preserved under convolution, the inter-start times of cycles are a proper random variable. Hence the associated renewal counting process,  $C(t; T)$  for any finite  $T$  has the limit [19]:

$$\text{As } t \rightarrow \infty, C(t; T) \rightarrow \infty \text{ with probability one.} \quad (8)$$

Equation (8) holds even when the mean inter-query time,  $E[X_i]$ , is infinite.

Let  $N_i(T)$  denote the number of cache hits in cycle  $i$ , given that the TTL is  $T$ . Note that  $N_i(T)$  has the same distribution as the number of cache hits in cycle 1, which in the notation of the inter-query renewal process is  $N(t) |_{t=T}$ . In particular, the expectations are equal:

$$E[N_i(T)] = E[N(T)] \quad (9)$$

From the definitions,

$$\begin{aligned} \text{The number of queries (hits plus misses) in the cycle } i \\ = N_i(T) + 1 \end{aligned}$$

The number of queries in cycles completed over  $(0, t]$

$$\begin{aligned} &= \sum_{i=1}^{C(t;T)} (N_i(T) + 1) \\ &= \sum_{i=1}^{C(t;T)} N_i(T) + C(t;T) \end{aligned}$$

Let

$$\begin{aligned} H(t; T) &= \text{hit ratio for cycles completed over} \\ &\quad \text{the interval } (0, t], \text{ given the TTL is } T \\ M(t; T) &= \text{miss ratio for cycles completed over} \\ &\quad \text{the interval } (0, t], \text{ given the TTL is } T \end{aligned}$$

From the definitions,

$$H(t; T) = \frac{\sum_{i=1}^{C(t;T)} N_i(T)}{\sum_{i=1}^{C(t;T)} N_i(T) + C(t; T)} \quad (10)$$

$$M(t; T) = \frac{C(t; T)}{\sum_{i=1}^{C(t;T)} N_i(T) + C(t; T)} \quad (11)$$

$$H(t; T) + M(t; T) = 1 \quad (12)$$

Let

$$\begin{aligned} H(T) &= \text{limiting hit ratio, } \lim_{t \rightarrow \infty} H(t; T) \\ M(T) &= \text{limiting miss ratio, } \lim_{t \rightarrow \infty} M(t; T) \end{aligned}$$

From Equation (10)

$$\begin{aligned} H(T) &\equiv \lim_{t \rightarrow \infty} H(t; T) \quad (13) \\ &= \lim_{t \rightarrow \infty} \frac{\sum_{i=1}^{C(t;T)} N_i(T)}{\sum_{i=1}^{C(t;T)} N_i(T) + C(t; T)} \\ &= \lim_{t \rightarrow \infty} \frac{\frac{1}{C(t;T)} \sum_{i=1}^{C(t;T)} N_i(T)}{\frac{1}{C(t;T)} \sum_{i=1}^{C(t;T)} N_i(T) + 1} \\ &= \frac{E[N(T)]}{E[N(T)] + 1} \text{ with probability one,} \quad (14) \end{aligned}$$

where the last equality is the result reported in the theorem and follows from applying the strong law of large numbers to  $\frac{1}{n} \sum_{i=1}^n N_i(T)$ , and recalling Equations (8) and (9) and noting that the function  $f(x) \equiv \frac{x}{x+1}$  is continuous for  $x > 0$ .

Note that for any finite  $t$ ,  $H(t; T)$  is a random variable (that is a function of various random variables), with a not-easily-determined distribution, while the limit is a constant. All probability is in a single point mass.

The derivation for the limiting miss ratio  $M(T)$  is analogous.